This question paper contains 5 printed pages]

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S. No. of Question Paper : 6716

Unique Paper Code : 32371501 HC

Name of the Paper : **Stochastic Processes and Queuing Theory**

Name of the Course : **B.Sc. (Hons.) STATISTICS**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Section I is *compulsory*.

Attempt *four* more questions, selecting *two* questions from each of the Sections II and III.

Use of simple calculator is allowed.

Section I

1. Attempt any *five* parts : $5 \times 3 = 15$

(*a*) In the classical ruin problem, what will be the effect of reducing the unit of stake from one to half, on the probability of ruin of the gambler ?

(*b*) If $\{N(t), t \geq 0\}$ is a Poisson process, then show the autocorrelation coefficient between $N(t)$ and $N(t+s)$ is $\sqrt{\frac{t}{t+s}}$.

P.T.O.

(*c*) Define convolution of two sequences $\{a_k\}$ and $\{b_j\}$, where $a_k = P(X = k)$ and $b_j = P(Y = j)$ with X and Y being two non-negative, integral valued random variables. Find the probability generating function of the sum of two independent random variables.

(*d*) Let $X(t) = A_0 + A_1 t + A_2 t^2$, where A_i, $i = 0, 1, 2$ are uncorrelated random variables with mean 0 and variance 1. Is $\{X(t), t \in T\}$ covariance stationary ?

(*e*) Define the following :

(*i*) Closed set

(*ii*) Ergodic state.

(*f*) Let $\{X_n, n \geq 0\}$ be a Markov chain having state space $S = \{1, 2\}$ with transition matrix

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Find the stationary distribution of given t.p.m.

(*g*) Let X_n, for n even, takes values +1 and –1 each with probability $\frac{1}{2}$, and for n odd, take values $\sqrt{a}, \frac{-1}{\sqrt{a}}$, with probabilities $\frac{1}{1+a}, \frac{a}{1+a}$ respectively (a is real number > -1 and $\neq 0, 1$). Further let X_n's be independent. Is $\{X_n, n \geq 1\}$ strictly stationary ?

Section II

2. (*a*) Let a fair coin be tossed indefinitely. Find the probability that two or more consecutive heads will not occur in n tosses. Also find the generating function of this event.

(*b*) Let $S_N = X_1 + X_2 + \ldots + X_N$, where N has Poisson distribution with mean *a*. If X_i's have *i.i.d.* Bernoulli distribution with $P(X_i = 1) = p$ and $P(X_i = 0) = 1 - p = q$, then show that :

(*i*) S_N has Poisson distribution with mean *ap*.

(*ii*) The joint distribution of S_N and N has the probability mass function

$$\Pr(N = n, S_N = y) = \frac{e^{-a} a^n p^y q^{n-y}}{y!\,(n-y)!} \text{ for } n = 0, 1, 2, \ldots; y = 0, 1, 2, \ldots n$$

(*iii*) Cov $(N, S_N) = ap$. 7,8

3. (*a*) Obtain probability generating function of the random variable X having the mass function.

$p_k = \frac{1}{(2)} q^{(|k|-1)}(1-q)$; $k = \ldots, -3, -2, -1, 1, 2, 3, \ldots$ where $0 < q < 1$.

(*b*) Six boys Dev (D), Hemant (H), Jeet (J), Mohan (M), Sunil (S) and Tejas (T) play the game of catching a ball. If D has the ball, he is equally likely to throw it to H, M, S and T. If H gets the ball, he is equally likely to throw it to D, J, S, T. If S has the ball, he is equally likely to throw it to D, H, M and T. If either J or T gets the ball, they keep throwing it to each other. If M gets the ball, he runs away with it. Obtain the transition probability matrix and classify the states. 7,8

4. (*a*) Define a persistent state and a transient state. Show that the state *j* is persistent iff

$$\sum_{n=0}^{\infty} p_{jj}^n = \infty$$

(*b*) Consider a Markov chain $\{X_n, n \geq 0\}$ with states 0 and 1 having transition probability matrix:

$$\begin{array}{c} X_{n-1} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} 1-(1-c)p & (1-c)p \\ (1-c)(1-p) & (1-c)p+c \end{pmatrix} \end{array} ; \quad 0 < p < 1;\ 0 \leq c \leq 1$$

with initial distribution $P\{X_0 = 1\} = p_1 = 1 - P\{X_0 = 0\}$. Show that correlation coefficient Corr $\{X_{n-k}, X_n\} = c^k$ for $0 < c < 1$. 7,8

Section III

5. (*a*) Divide the interval $[0, t]$ into a large number *n* of small intervals of length *h* and suppose that in each small interval, Bernoulli trials with probability of success λh and with probability of failure $(1 - \lambda h)$ are held. Show that the number of successes in an interval of length *t* is a Poisson process with mean λt. State the assumptions you make.

(*b*) Show that for the linear growth process, the second moment $M_2(t)$ satisfies the differential equation $M_2'(t) = 2(\lambda - \mu)M_2(t) + (\lambda + \mu)\, M(t)$. Further, show that variance is

$$\text{Var}\,(X(t)) = i\frac{\lambda+\mu}{\lambda-\mu} e^{(\lambda-\mu)t}\,(e^{(\lambda-\mu)t} - 1);\ \lambda \neq \mu$$

where *i* is the population size at $t = 0$. 7,8

6. (*a*) Define a Poisson process. State the postulates under which a count process will be a Poisson process.

(*i*) Show that random selection from a Poisson process yields a Poisson process.

(*ii*) If $N_1(t)$, $N_2(t)$ are two independent Poisson processes with parameters λ_1 and λ_2 respectively, then obtain the distribution of $N(t) = N_1(t) - N_2(t)$.

(*b*) Describe the classical ruin problem. Derive an expression for the expected duration of the game, which is finite. Obtain the limiting expression as $a \to \infty$. 7,8

7. (*a*) In the case of (M/M/1); (N/FCFS) queuing model, derive the steady-state probability distribution and obtain the expressions for :

(*i*) Expected number of customers in the system

(*ii*) Expected number of customers in the queue

(*iii*) Expected waiting time in the system.

(*b*) A super market has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate :

(*i*) the probability that the cashier is idle

(*ii*) the average number of customers in the queue

(*iii*) the average time a customer spends in the system. 7,8

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6717**

Unique Paper Code : **32371502** **HC**

Name of the Paper : **Statistical Computing using C/C++ Programming**

Name of the Course : **B.Sc (H) Statistics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Attempt any *ten* parts : $10 \times 3 = 30$

(*i*) State whether the following statements are true *or* false :

(*a*) The **do-while** statement first executes the loop body and then evaluates the loop control expression.

(*b*) The **default** case is required in the switch statement.

(*c*) The return type of a function is int by default.

(*d*) Parentheses can be used to change the order of evaluating expressions.

P.T.O.

(*e*) The underscore can be used anywhere in an identifier.

(*f*) An integer can be added to a pointer.

(*ii*) Fill in the blanks :

(*a*) A program start execution from.....function.

(*b*) The.....statement when executed in a switch statement causes immediate exit from the structure.

(*c*) The.....specification is used to read or write short integer.

(*d*) The.....operator returns the number of bytes the operand occupies.

(*e*) The keyword.....can be used to create a data type identifier.

(*f*) The escape sequence character.....causes the cursor to move to the next line on the screen.

(*iii*) float x = –4.2, xmin = 4.7;

if (abs(x) <xmin) x = (x>0) ? xmin : -xmin;

printf("%f", x);

what will be the output of the above code ?

(*iv*) What is a structure ? How does a structure differ from an array ?

(*v*) Describe *two* different ways to access an array element.

(*vi*) Consider the program segment to answer the following. In this case, assume that the memory addresses of x as 100, y as 300 and u starting from 700


```
double x=20.5, y=10.5, z;
double *px, *py;
int u[3][3] = {{1, 11, 111}, {2, 22, 222}, {3, 33, 333}};
int *v;
px =&x;
py =&y;
v =&u[1][1];
z = (*v + 1)*(*px – y)/2;
```

(*a*) What is the value of *px and z ?

(*b*) What is the value of *(v-1)* *(v-4) ?

(*vii*) Write a loop that will generate every third integer, beginning with $i = 2$ and counting for all integers that are less than 100. Calculate the sum of those integers that are divisible by 5.

(*viii*) Write a conditional expression for the following :

If the variable divisor is not zero, divide the variable dividend by divisor and store the result in variable quotient. If the divisor is zero, assign it to the quotient.

(*ix*) Given that int $x = 2$, $y = 3$, $z = 2$, $t = -4$; evaluate the following expressions :

(*a*) $z - (x + z)\%2 + y$

(*b*) $x! = z \&\&! (y < z) \| x > t$

(*x*) What are function prototypes in C ? What is their purpose ? Illustrate with example.

(*xi*) Define a self-referential structure contacting the following three members :

(*a*) A 40 element character array called name

(*b*) An integer quantity called lost

(*c*) A floating point quantity called percent

Include the tag team within the structure definition.

(*xii*) Find error(s) in the following program :

```
#include<stdio.h>
main();
 {
      int 9x =2, y;
 scanf("%d",y);
      putchar(\n);
      printf('%c', "A");
      return(0);
 }
```

2. Write the output of any *two* parts from the following : 2×5=10

(*i*)

```
#include<stdio.h>
int a = 17;
main()
{
```



```
int a=5, b=12, x = 15, y = 2, z = -32765, t =100;
    float r, s;
    r = x>y ? x/y : x*y;
    s = z + 5;
 b +=a;
 a = b - a;
 b = b - a;
 printf("r = %f\n s=%f", r, s);
 printf("a = %d \n b=%d", a, b);
 printf("%d\n", 10 + ++t);
 return (0);
}
```

(*ii*)

```
#include<stdio.h>
int func1(inta);
int func2(inta);
main()
 {
inta=0,b=1, count;
for(count = 1; count <=5; count++)
     {
        b+=func1(a) + func2(a);
        printf("%d", b);
```

```
    }
}
intfunc1(inta)
{
intb;
b=func2(a);
 return (b);
 }
int func2(int a)
{
 static int b;
 b+=1;
 return (b+a);
}
```

(*iii*)

```
#include<stdio.h>
main()
{
inta, b, *p1, *p2, x, y, z;
a = 12;
b = 4;
p1 = &a;
p2 = &b;
```



```
x = *p1 * * p2 - 6;
y = 4* - *p2 / *p1 + 10;
printf("a =%d, b =%d\n", a, b);
pinrtf("x = %d, y = %d\n", x, y);
*p2 = *p2 + 3;
*p1 = *p2 - 5;
z = *p1 * *p2 - 6;
printf("a = %d, b = %d\n", a, b);
printf("z = %d\n", z);
return (0);
}
```

3. Attempt any *two* parts : 2×5=10

(*i*) What is a pointer ? How can it be initialized ? Also, discuss how initial values can be assigned to two-dimensional arrays with the help of examples.

(*ii*) Describe different forms of loop available in C. How would you decide the use of one of the three loops in C for a given problem ?

(*iii*) Distinguish between the following with the help of examples :

(*a*) Global and local variables

(*b*) Actual and formal arguments

4. Attempt any *two* parts : 4½ × 2 = 9

(*i*) Write a C-program to calculate the product of two matrices A and B of order $m \times n$ and $n \times p$ respectively.

(*ii*) Write a C-program to fit Poisson distribution to the following data :

x :	0	1	2	3	4	5
f :	109	65	22	7	3	1

(*iii*) Write a C program to compute the roots of quadratic equation $ax^2 + bx + c = 0$.

5. Attempt any *two* parts : 2×8=16

(*i*) In an experiment on immunization of cattle from tuberculosis, the following results were obtained :

	Affected	Unaffected
Inoculated	12	28
Not inoculated	13	7

Write a C-program to test whether vaccine is effective in controlling the incidence of the disease.

(*ii*) Develop a function to draw a random sample of size n from gamma distribution with parameters k and λ. Also find its mean and variance. Hence write a C-program to perform the above mentioned tasks using files.

(*iii*) Develop a function to calculate correlation coefficient for the data given on r.v.s X and Y. Hence, using the function, develop a program to compute multiple correlation coefficient of X on r.v.s Y and Z.